CBCS SCHEME

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18MAT21

Second Semester B.E. Degree Examination, July/August 2021
Advanced Calculus and Numerical Methods

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. Find the divergence and curl of the vector, $\vec{V} = (xyz)i + (3x^2y)j + (xz^2 - y^2z)k$ at the point (2, -1, 1). **(06 Marks)**

b. Find the workdone in moving a particle in the force field $F = 3x^2i + (2xz - y)i + zk$ along the curve defined by $x^2 = 4y$, $3x^3 = 8z$ from x = 0 to x = 2. **(07 Marks)**

c. Evaluate the surface integral $\iint_S \vec{F}.\vec{N}ds$ where $\vec{F} = 4xi - 2y^2j + z^2k$ and S is the surface bounding the region $x^2 + y^2 = 4$, z = 0 and z = 3. **(07 Marks)**

2 a. Find Curl (Curl $\vec{A}$) where $\vec{A} = x^2yi - 2xzj + 2yzk$ at the point (1, 0, 2). **(06 Marks)**

b. If $\vec{u} = x^2i + y^2j + z^2k$ and $\vec{v} = yzi + zxj + xyk$, show that $\vec{u} \times \vec{v}$ is solenoidal. **(07 Marks)**

c. Evaluate $\int_C (\sin zdx - \cos xdy + \sin ydx)$ by using Stoke's theorem, where C is the boundary of the rectangle $0 \le x \le \pi$, $0 \le y \le 1$ and z = 3. **(07 Marks)**

3 a. Solve: $(D^4 - 1)y = 0$ **(06 Marks)**

b. Solve: $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 4y = \sinh(2x+3)$ by Inverse differential operator method. **(07 Marks)**

c. Solve: $x^3\frac{d^3y}{dx^3} + 3x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = x + \log x$ **(07 Marks)**

4 a. Solve: $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = (1 - e^x)^2$ **(06 Marks)**

b. Solve: $(D-2)^2y = 8(e^{2x} + x + x^2)$ by Inverse differential operator method. **(07 Marks)**

c. A particle moves along the x-axis according to the law $\frac{d^2x}{dt^2} + \frac{6dx}{dt} + 25x = 0$. If the particle is started at x = 0 with an initial velocity of 12ft/sec to the left, determine x(t). **(07 Marks)**

5 a. Form the partial differential equation by eliminating the arbitrary constants in $(x-a)^2 + (y-b)^2 = z^2\cot^2\alpha$, where α is the parameter. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ for which $\frac{\partial z}{\partial y} = -2\sin y$ when x = 0 and z = 0 if y is an odd multiple of π/2. **(07 Marks)**

c. Derive one dimensional heat equation. **(07 Marks)**

6 a. Form the partial differential equation by eliminating the arbitrary functions from $Z = f(x + at) + g(x - at)$. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial y^2} = z$ given that when $y = 0$, $z = e^x$ and $\frac{\partial z}{\partial y} = e^{-x}$. **(07 Marks)**

c. Solve $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 2(x + y)u$, by the method of separation of variables. **(07 Marks)**

7 a. Find the nature of the series $\sum_{n=1}^{\infty} a^{n^2} x^n$, $a < 1$ **(06 Marks)**

b. Prove that: $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ **(07 Marks)**

c. If $x^3 + 2x^2 - x + 1 = aP_0(x) + bP_1(x) + cP_2(x) + dP_3(x)$ find the values of a, b, c, d. **(07 Marks)**

8 a. Test for convergence the series,

$$\frac{1^2}{2} + \frac{2^2}{2^2} + \frac{3^2}{2^3} + \frac{4^2}{2^4} + \ldots\ldots\ldots$$ **(06 Marks)**

b. Express $x^3 + 2x^2 - 4x + 5$ interms of Legendre polynomials. **(07 Marks)**

c. Show that i) $P_2(\cos\theta) = \frac{1}{4}(1 + 3\cos 2\theta)$ ii) $P_3(\cos\theta) = \frac{1}{8}(3\cos\theta + 5\cos 3\theta)$. **(07 Marks)**

9 a. From the following table of half yearly premium for policies maturing at different ages, estimate the premium for policies maturing at age of 46. **(06 Marks)**

Age	45	50	55	60	65
Premium (In Rupees)	114.84	96.16	83.32	74.48	68.48

b. Find cube root of 37 correct to 3 decimal places, using Newton-Raphson method. **(07 Marks)**

c. Use Simpson's 1/3rd rule to find $\int_0^{0.6} e^{-x^2} dx$ by taking 6 sub-intervals. **(07 Marks)**

10 a. Using Newton's backward Interpolation formula, find the interpolating polynomial function given by the following table:

x	10	11	12	13
f(x)	22	24	28	34

(06 Marks)

b. Find a Real Root of the equation $x^3 - 2x - 5 = 0$ correct to three decimal places using Regula Falsi method. **(07 Marks)**

c. Evaluate $\int_0^1 \frac{x\,dx}{1+x^2}$ by Weddle's rule taking seven ordinates. **(07 Marks)**

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